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# COMMENT ON “DISCRETIZATION PROBLEMS OF FUNCTIONAL INTEGRALS IN PHASE SPACE”

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## Comment on "Discretization problems of functional integrals in phase space"

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It is shown that the definition of path integrals without limiting procedure does not give rise to unresolved factor-ordering ambiguities. Any inference from the recent work of Langouche *et al.* [Phys. Rev. D **20**, 419 (1979)] that one is forced to resort to "discretization" (time slicing) to resolve these so-called "ambiguities" is thereby shown to be incorrect.

Recently, Langouche, Roekaerts, and Tirapegui (LRT) claimed<sup>1</sup> that "the definition of functional integrals without limiting procedure ... has exactly the same ambiguities as the definition through discretization" and that "the way to eliminate the ambiguity is precisely by using ... discretization." The purpose of this note is to examine these statements.

The basis for the first assertion is presumably a paper I wrote for the 1978 Marseille Colloquium on Feynman Path Integrals<sup>2</sup> (Ref. 9 in LRT's paper). I will first summarize the result of this paper and then I will interpret it.

The problem in Ref. 2 was to find a path-integral solution  $K$  of the Schrödinger equation, i.e.,

$$[\underline{H}(P, Q, t) - i\hbar\partial/\partial t_b]K(q_b, t_b; q_a, t_a) = 0, \quad (1)$$

such that  $K = \delta(q_b - q_a)$  as  $t_b - t_a$ .  $\underline{H}$  is an arbitrary Hamiltonian operator (no restrictions whatsoever on its form or properties), and, naturally,  $P$  is  $-i\hbar\partial/\partial q_b$  and  $Q$  is  $q_b$ . When the path-integral solution was obtained using the formulation with no limiting procedure<sup>3</sup> (LRT's Ref. 6) pioneered by DeWitt-Morette,<sup>4</sup> one was faced with undefined path integrals involving the coupling of position and momentum terms at the same time  $t$ . These path integrals have the generic form (in one dimension)

$$J = \int_{\Phi} q^m(t)p^n(t)dw(p, q), \quad (2)$$

where  $\Phi$  is phase space and  $w$  is the "measure" used. This "ambiguity" is as it should be because the specific way in which the  $P$  and  $Q$  terms were ordered in  $\underline{H}$  had not yet been considered. No definition of the path integral can possibly create missing information. This point is misunderstood by many authors. When the specific ordering is considered, the correct result is obtained. For the path integral  $J$  in Eq. (2), this is done as follows:

(1) First, define the classical function  $H_c \equiv \underline{H}(P = p, Q = q, \hbar = 0)$ . This is generally the "classical Hamiltonian."

(2) Next, turn the function  $q^m p^n$  appearing in  $J$

into an operator by ordering the factors according to one of the correspondence rules<sup>5</sup> connecting the function  $H_c$  to the operator  $\underline{H}$ . For definiteness, put the resulting operator in normal-ordered form by repeated use of  $\underline{Q}\underline{P} - \underline{P}\underline{Q} = i\hbar$ , to get something of the form

$$\sum_{i=0}^{m+n} a_i(\hbar) \underline{Q}^i \underline{P}^{m+n-i}.$$

(3) Then  $J$  must be taken to be

$$J = \sum_{i=0}^{m+n} a_i(\hbar) \lim_{t-t' \rightarrow 0^+} \int_{\Phi} q^i(t) p^{m+n-i}(t') dw(p, q)$$

(a well-defined path integral since  $t \neq t'$ ) if the Schrödinger equation (1), with the boundary condition, is to be satisfied.

A very rough summary of this procedure is as follows: if  $\underline{H}$  contains a term  $\underline{P}\underline{Q}\underline{P}$  giving rise to an undefined path integral  $\int_{\Phi} p^2(t)q(t)dw(p, q)$ , then this path integral must be replaced by

$$\lim_{t-t' \rightarrow 0^+} \lim_{t-t'' \rightarrow 0^+} \int_{\Phi} p(t)q(t')p(t'') dw(p, q)$$

(a well-defined path integral since  $t \neq t' \neq t''$ ). Alternatively, since  $\underline{P}\underline{Q}\underline{P} \approx \underline{Q}\underline{P}^2 - i\hbar\underline{P}$ , the same result is obtained if one replaces the undefined path integral by

$$\lim_{t-t' \rightarrow 0^+} \int_{\Phi} q(t)p^2(t') dw(p, q) - i\hbar \int_{\Phi} p(t) dw(p, q)$$

(the equivalence of these alternative forms is proved in Ref. 2). Reference 2 contains a very detailed proof that this procedure leads to expressions that are solutions of the Schrödinger equation (1), with the boundary condition, regardless of  $\underline{H}$ ; no specific form for  $\underline{H}$  need be assumed. This proof emphasizes the fact that this is no mere "prescription" (LRT's word, p. 429) known to give correct answers where it has been applied, but rather a general, consistent procedure.

Thus, the "ambiguity" is resolved in a natural manner as soon as the missing information on  $\underline{H}$  is supplied. No one denies that it might be pos-

sible to give meaning to undefined path integrals within the usual "discretization" definition of path integrals, or even that one mechanism for that is not far from a "discretized" version of my own procedure (as LRT seem to suggest on p. 429 of their paper). The reader might conclude that the choice between "discretization" and "no limiting procedure" is a matter of taste and elegance; I believe the definition without limiting procedure is favored for reasons beyond taste and elegance (reasons given in Ref. 2), but that is not the issue here. The point is that the definition of path integrals without limiting procedure does *not* give rise to un-

resolved factor-ordering ambiguities. Any inference from LRT's work that one *must* resort to the discretization definition to resolve these so-called ambiguities is thereby incorrect.

Finally, let us note that a consensus seems to be emerging on the fact that no definition of path integrals will reveal *which* quantum operator ("factor ordering"<sup>6</sup>) one must associate to a given classical Hamiltonian, but that once the choice of operator is made, path integrals *can* be unambiguously defined to yield the correct propagator.

<sup>1</sup>F. Langouche, D. Roekaerts, and E. Tirapegui, Phys. Rev. D **20**, 419 (1979) (statements on pp. 419 and 421).

<sup>2</sup>Maurice M. Mizrahi, in *Feynman Path Integrals*, proceedings of the International Colloquium, Marseille, 1978 (Lecture Notes in Physics), edited by S. Albeverio *et al.* (Springer, Berlin, 1979), Vol. 106, pp. 234-53. (Note that some of the pages were interchanged, as evidenced by the numbering of the equations.)

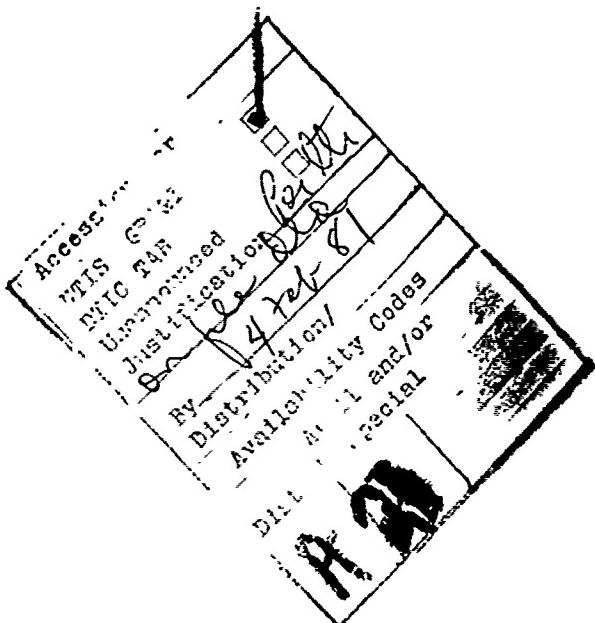
<sup>3</sup>Maurice M. Mizrahi, J. Math. Phys. **19**, 298 (1978);

**21**, 1965 (E) (1980).

<sup>4</sup>See, e.g., C. DeWitt-Morette, A. Maheshwari, and B. Nelson, Phys. Rep. **50C**, 256 (1979).

<sup>5</sup>In Ref. 2, extensive use was made of Cohen's scheme for correspondence rules [L. Cohen, J. Math. Phys. **7**, 781 (1966)].

<sup>6</sup>Note that some correspondence rules, e.g.,  $\rho \rightarrow e^{iQ/4}Pe^{iQ/4}$ , are not obtained from factor ordering, a point discussed in Ref. 2.



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